

PH.D. DISSERTATION EVALUATION REPORT

INSTITUTION: AGH-UST Krakow, Department of Applied Mathematics

DISSERTATION: **Decompositions of Some Classes of Graphs...**

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The dissertation is very well written and brings new fundamental results, which significantly advance research in graph decompositions.

Graph decomposition is one of the classical topics in graph theory, which has been intensively studied in modern graph theory for more than fifty years, and whose origins go back to Kirman's results from 1847. Graph labelings, introduced by Rosa in 1967 and further enhanced by many other researchers, proved to be a very efficient tool in this area.

First two chapters contain a comprehensive introduction to the topic and thoroughly cover the classical and newer labelings. Chapter 3 then deals with what I consider the core of this dissertation, namely labelings of tripartite graphs. In particular, the ρ -tripartite labeling (which the candidate discovered with co-authors) and especially the 1-rotational ρ -tripartite labeling (which is solely the candidate's contribution) present an extremely useful tool for decompositions of complete graphs K_{2nk+1} and K_{2nk} , respectively, into graphs with n edges.

To illustrate the strength of the 1-rotational ρ -tripartite labeling, I can offer a personal experience. For more than one year, I had been working with one of my students on decompositions of complete graphs K_{16k+1} and K_{16k} into certain class of graphs on eight edges. The complete solution in a very raw form took about 50 pages and we were worried whether such a result in its final form would be appropriate for publication as a single paper, as it would easily take at least 60 pages. Then I attended the candidate's talk at The Fourth Kliakhandler Conference on Graph Decompositions and learned about the new labeling, introduced by Ryan. Although we had to re-do the whole paper, we were happy to do it, as the labeling allowed us to shorten the complete solution to about ten pages.

Chapter 4 deals with another new and useful tool, alpha-accommodations. This technique allows decomposition of disconnected graphs when the components or in general subgraphs of the given graph admit some currently known labelings with some additional properties.

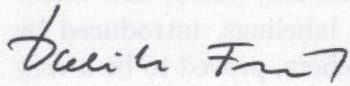
Chapter 5 then expands the methods to hypergraphs, and builds, besides others, on recent results by the candidate's Supervisor M. Meszka and the founder of the field, A. Rosa.

Finally, Chapter 6 lists some particular applications of the new labelings into important specific classes of graphs like unicyclic graphs with one or two vertices of degree three and unions of cycles.

Overall, the dissertation is of high quality and I have no doubts that it can be successfully defended. I wish every dissertation I have or will have been evaluating would be such a pleasant and enjoyable reading, and bring such useful new tool as this one.

Without any hesitation, I recommend the dissertation be accepted, and the candidate awarded the degree of Doctor of Philosophy in Mathematics with Distinction.

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