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REPORT FOR THE PHD THESIS

title of the thesis: *Decompositions of Some Classes of Graphs Through Vertex Labelings*
submitted by: Ryan C. Bunge

The dissertation is devoted to study and combine two highly-investigated topics in Graph Theory: the concept of Rosa-type labelings (Rosa, 1967) and the problem of decomposing a graph into edge-disjoint copies isomorphic to some subgraphs, closely related to the well-known concepts of Kirkman triple systems (1850) and Steiner triple systems (1853), and the Oberwolfach problem (Ringel, 1967). Since a number of several results (by R.C. Bunge) has been presented – from those chronologically earliest to those generalizing many previous or already known – and several different proof techniques have been used – both elementary and involved ones – in what follows, I will only discuss a few of them (those I find most interesting).

GENERAL OVERVIEW

In Section 2, extensions of Rosa-type labelings are presented. In particular, the Author introduces the concept of λ-fold Rosa-type labelings which then are used to establish the relevant nice results for the existence of purely cyclic decomposition of the complete multigraph ${}^{\lambda}K_{n+1}$ (Theorems 2.4 and 2.5). In addition, the case of circulant graphs is studied, following the existing problems in the literature.

- Section 2.2: Why λ-fold β-labelings are not considered?
- Proof of Theorem 2.4, page 14, lines 13-14:
  "Since G' ... $(K_{2k+1} - I, G')$-design". I find this not obvious.

In Section 3, the concept of α-labelings (Rosa, 1967) is extended to tripartite graphs, by introducing the definitions of σ- and ρ-tripartite labelings (Theorems 3.3 and 3.5), and to multigraphs (Theorems 3.6 and 3.7), respectively. The proofs of these results possess a nice hierarchical structure, based upon clever case analysis, each of them supported by the already discussed results. In addition, circulant graphs are studied as well as 1-rotatinal variants of ρ-tripartite labeling.

- Page 24, line 3:
  Is the function h a one-to-one function? G is assumed to have n edges.
  Similarly on page 27, line 10.
- Page 36, line 4 in the proof of Theorem 3.8:
  Clearly, the sum of Δ_i's is A, but is this obvious that clicking the last component in Δ_i results in a component in Δ_{i+1}?

In Section 4, a novel concept of accommodating and, respectively, α-accommodating, a graph by another graph is proposed, given their relevant vertex-labelings, which allows to establish the existence of G-decompositions where G is a member of an infinite family of graphs. In this concept, the crucial role is played by the Fundamental Construction (p. 48), and, among other interesting new results, let me on mention Theorem 4.10 which settles the existence of a (K_{2nt+1},G)-design for any positive integer t, where G is a 2-regular almost bipartite graph with n-edges, thus generalizing some results established by Blinco et al., in 2004.

- The part of the proof of Theorem 4.10 is based upon Lemmas 6.8-6.10, which are unnaturally postponed to be presented in Section 6.

Decomposition of uniform hypergraphs, closely related to Steiner systems $S(k,m,n)$, is studied in Section 5. This section, ample in preliminary results and nice examples, includes relatively less crucial results (compared to previous sections). Nevertheless, I find three things that are noteworthy: exploiting the concept of r-pyramidal action, a non-trivial open problem (Conjecture 5.3, Bunge et al. 2018), and Theorem 5.4 which establishes the necessary and sufficient condition for the existence of the LS_3-decomposition of the complete 3-uniform hypergraph $K_n^{(3)}$, where LS_3 is a 3-uniform linear 3-star.

- What is the reason for focusing only on 3-uniform linear 3-stars?

Finally, Section 6 gathers several results concerning the labelings described in Sections 1-5. In particular, by an involved case-by-case proof, the existence of a 1-rotational G-decomposition of the complete graph K_{2nt} is established, where G is a simple graph resulting from adding an edge to an n-vertex path, and $G \neq C_n$ (Theorem 6.6). In addition, a sequence of lemmas is provided which are devoted to α-accommodating ρ-tripartite labelings of 2-regular graphs; some of these results have been exploited in Section 4, in the proof of Theorem 4.10.

OTHER REMARKS

Even though the thesis was written in a correct way, there are some typo-graphical errors to correct. In a few places, I also find the style hard to follow, but subjectively, since I am not a native speaker. In addition, I feel that the results could be presented in a more comparative manner (to the already established results and used proof techniques of others, i.e., which of the used proof techniques are just known, modified, or completely new). Finally, I am missing short subsections devoted to the algorithmic aspect of the considered variants (thus following a short note in Section 1), and a kind of summary-type ending section would be also useful. Of course, these shortcomings do not substantially affect the quality of the work and do not disqualify it in any way.

- In Theorem 2.4, G should be assumed to be simple, I think.
- Proof of Theorem 2.4, page 14, line 3:
  "... cyclic (K_{2k+1},G')-design" should be completed with "by Theorem 1.1", I think.
  Similarly in the proof of Theorem 2.5, page 14, line -2.
- Page 45, line -5, in (a3): $\cap$ ---?--> $\cup$

SUMMARY

In my opinion, the contribution of this dissertation meets and greatly exceeds the formal requirements for a doctoral thesis. Thereby, taking into account the following aspects (although slightly shadowed by the low number N of citations and the h-index: N=7 and h=1 [Web of Science, 29.06.2019, 11:00 AM]):

- the ability of the Candidate to generalize several results/definitions/concepts;
- the scientific novelty and an extensive range of research;
- a meritorious presentation and the discussion of obtained results;
- the scientific activity, confirmed by several papers published in commonly known and highly ranked scientific journals (Ars Combinatoria, Discrete Mathematics, Journal of Graph Theory, and Utilitas Mathematica, just to mention a few);
- the ability of the Candidate to cooperate, with a very high contribution to each of these joint works, and even mentoring some participants in the research,

I would like also to propose to reward Ryan C. Bunge's doctoral dissertation.

29.06.2019
(place and date)

Paweł Gładki
(signature of referee)

SHORT EVALUATION REPORT FOR THE PHD THESIS

title of the thesis: Decompositions of Some Classes of Graphs Through Vertex Labelings
submitted by: Ryan C. Bunge
referee: dr hab. Paweł Żyliński, prof. UG

	YES	NO
A) I recommend the thesis to Department of Discrete Mathematics, Faculty of Applied Mathematics, at the AGH University of Science and Technology in its present form.	☒	☐
B) Small changes have been agreed with the candidate and should be taken into account.	☐	☒
C) Revision of the thesis is necessary.	☐	☒

	HIGH	MEDIUM	LOW
D) How do you evaluate the quality of the content of the thesis (depth and breadth)?	☒	☐	☐
E) How is the quality of presentation?			
(i) mathematical presentation	☒	☐	☐
(ii) textual presentation	☒	☐	☐
(iii) images	☒	☐	☐
F) What is the degree of originality?	☒	☐	☐
G) How do you evaluate the thesis with respect to the further development of the field?	☒	☐	☐
H) How do you assess the independence/self-dependance of the candidate?	☒	☐	☐

Evaluation Summary

VERY GOOD	GOOD	SATISFACTORY	SUFFICIENT	REJECT
☒	☐	☐	☐	☐

29.06.2021
(place and date)

Paweł Żyliński
(signature of referee)